12/2 Roll No. ……………………..

PHY–501

Mathematical Physics and Classical Mechanics

M. Sc. PHYSICS (MSCPHY–12/13/16)

First Year, Examination, 2017

Time : 3 Hours **Max. Marks : 80**

Note : This paper is of **eighty (80)** marks containing **three (03)** Sections A, B and C. Learners are required to attempt the questions contained in these Sections according to the detailed instructions given therein.

Section–A

(Long Answer Type Questions)

Note : Section 'A' contains four (04) long answer type questions of nineteen (19) marks each. Learners are required to answer *two* (02) questions only.

1. Describe recurrence formula for $\mathrm{H}_n(x)$ and to show that $\mathrm{H}_n(x)$ is a solution of Hermite equation. Also find the value of $\int_{-\infty}^{\infty} e^{-x^2} \mathrm{H}_2(x)\, \mathrm{H}_3(x)\, dx$.

2. (a) Express the functions :

$$f(x) = \begin{cases} l \text{ when} & |n| \leq l \\ 0 & |n| > l \end{cases}$$

as a Fourier integral.

Hence evaluate :

$$\int_0^{\infty} \frac{\sin\lambda \cos\lambda x}{\lambda}\, d\lambda$$

(b) Find the Laplace transform of (1 + sin 2*t*).

3. State and explain the Hamilton's principle of least action. Derive Lagrange's equation from Hamilton's principle. When string of a pendulum is elastic then set up Lagrange's equation for small oscillation where motion is considered to a vertical plane.
4. Derive an expression for the Gregory-Newton forward and backward difference interpolation formula.

Section–B

(Short Answer Type Questions)

Note : Section 'B' contains eight (08) short answer type questions of eight (08) marks each. Learners are required to answer *four* (04) questions only.

1. Show that :

$$\frac{d}{dx}\ \mathrm{J}_n^2 + \mathrm{J}_{n+1}^2 = 2\left(\frac{n}{x}\mathrm{J}_n^2 - \frac{n+1}{x}\mathrm{J}_{n+1}^2\right)$$

2. Explain the derivative of two-dimensional wave equation.
3. Show that $\frac{\partial \mathrm{A}_\lambda}{\partial x_\mu}$ is not a tensor although A_λ is a covariant tensor of rank one.
4. Surface as a sphere is a two-dimensional Riemannian space. Find the fundamental metric tensor.
5. Discus in brief equation of canonical transformation.
6. Explain Hamilton-Jacobi theory and show that the transformation $q = \sqrt{2}\sin \mathrm{Q}$ and $p = \sqrt{2}p\cos \mathrm{Q}$ is canonical.

7. Derive an expression for the Stirling interpolation formula.
8. Explain numerical solutions of ordinary differential equation with an example.

Section–C

(Objective Type Questions)

Note : Section 'C' contains ten (10) objective type questions of one (01) mark each. All the questions of this Section are compulsory.

1. The integral $\int_0^{n} P_n(\cos\theta)\sin 2\theta\, d\theta$, $n > 1$ where $P_n(x)$ is the Legendre polynomial of degree n equal to :

   (a) 1

   (b) $\frac{1}{2}$

   (c) 0

   (d) 2

2. If $J_{n+1}(x) = \frac{2}{x} J_n(x) - J_0(x)$, then n is :

   (a) 0

   (b) 2

   (c) –1

   (d) None of the above

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3. In the Fourier transform the value of $\int_0^{\infty} e^{-x^2} dx$ is :

(a) $\frac{\pi}{2}$

(b) $\frac{\sqrt{\pi}}{2}$

(c) $\frac{\pi}{\sqrt{2}}$

(d) $\sqrt{\frac{\pi}{2}}$

4. If L{F(t)} = $\overline{f}(s)$, then L{t F(t)} is :

(a) $\overline{f}'(s)$

(b) $-\overline{f}'(s)$

(c) $\overline{f}'(s)+\overline{f}(s)$

(d) $s\overline{f}'(s)+\overline{f}(s)$

5. If A^{μ} and B_{μ} components of contravarient and covariant tensors, what is the nature of the quantity $A^{\mu}B_{\mu}$:?

(a) A covariant tensor of rank 2

(b) Mixed tensor of rank 1

(c) Rank zero scale

(d) A mixed tensor of rank 2

6. The Christoffel symbols of the first kind $[ij, k]$ is :

(a) $\frac{1}{2}\left[\frac{\partial g_{jk}}{\partial x^i} - \frac{\partial g_{ik}}{\partial x^j} - \frac{\partial g_{ij}}{\partial x^k}\right]$

(b) $\frac{1}{2}\left[\frac{\partial g_{jk}}{\partial x^i} - \frac{\partial g_{ik}}{\partial x^j} + \frac{\partial g_{ij}}{\partial x^k}\right]$

(c) $\frac{1}{2}\left[\frac{\partial g_{jk}}{\partial x^i} + \frac{\partial g_{ik}}{\partial x^j} - \frac{\partial g_{ij}}{\partial x^k}\right]$

(d) $\frac{1}{2}\left[\frac{\partial g_{jk}}{\partial x^i} + \frac{\partial g_{ik}}{\partial x^j} + \frac{\partial g_{ij}}{\partial x^k}\right]$

7. In the case of canonical transformation :

(a) The form of the Hamilton equation is preserved.

(b) The form of Lagrange equations is preserved.

(c) Hamilton's principle is satisfied in old as well as in the new co-ordinates.

(d) The form of the Hamilton's equations may or may not be preserved.

8. The equation $\frac{d}{dt}\left(\frac{\partial L}{\partial q_i}\right) - \frac{\partial L}{\partial q_i} = 0$ where L = T – V is :

(a) Lagrangian equation for conservative system.

(b) Lagrangian equation for non-conservative system.

(c) Equation of motion of harmonic oscillator.

(d) None of these

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9. For Lagrange's interpolation formula if $f(0.5) = 4.56$ and $f(0.8) = 5.07$, the value of $f(0.55)$ is :

   (a) 4.645

   (b) 6.326

   (c) 4.000

   (d) 5.326

10. Runge-Kutta method for second order, first degree linear differential equation is :

   (a) $\dfrac{dy}{dx} = f\left(x, y, \dfrac{d^2y}{dx^2}\right)$

   (b) $\dfrac{d^2y}{dx^2} = f\left(x, y, \dfrac{dy}{dx}\right)$

   (c) $\dfrac{dy}{dx} = f\left(x, y, \dfrac{dy}{dx}\right)$

   (d) $\dfrac{d^2y}{dx^2} = f\left(x, y, \dfrac{dx}{dy}\right)$