## MAT-501

## Advanced Algebra M. Sc. MATHEMATICS (MSCMAT-12)

First Year, Examination, 2018
Time : 3 Hours
Max. Marks : 80
Note : This paper is of eighty (80) marks containing three (03) Sections A, B and C. Attempt the questions contained in these Sections according to the detailed instructions given therein.

## Section-A

(Long Answer Type Questions)
Note: Section 'A' contains four (04) long answer type questions of nineteen (19) marks each. Learners are required to answer two (02) questions only.

1. Define 'Direct product of two groups'. Let $\mathrm{G}_{i}$ ( $1 \leq i \leq n$ ) be $n$ groups and G is the external direct product of these groups, than each $g \in G$ can be written uniquely as product of elements from $\mathrm{G}_{1}, \mathrm{G}_{2}$,
$\qquad$ , $\mathrm{G}_{n}$.
2. Let $G$ be a finite group, then $o(\mathrm{G})=\sum_{a \in \mathrm{D}} \frac{\mathrm{o}(\mathrm{G})}{\mathrm{oN}(a)}=\sum_{i=1}^{n} \frac{\mathrm{o}(\mathrm{G})}{\mathrm{o}\left[\mathrm{N}\left(a_{i}\right)\right]}$, where D be a set of distinct elements $a_{1}, a_{2}, \ldots . . ., a_{n}$ taken from each of the conjugate classes of G.

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3. Define 'Real inner product space' and 'Norm of a Vector'. Let V be an inner product space. Then for arbitrary vectors $u, v \in \mathrm{~V}$ we have $\mid\langle u, v\rangle\|\leq\| u\| \| v \|$.
4. Let V be a finite dimensional inner product space and W be its subspace. Then V is direct sum of W and $\mathrm{W}^{\perp}$. Also show that $\operatorname{dim} W^{\perp}=\operatorname{dim} V-\operatorname{dim} W$.

## Section-B

## (Short Answer Type Questions)

Note : Section ' $B$ ' contains eight (08) short answer type questions of eight (8) marks each. Learners are required to answer four (04) questions only.

1. Let G be a group. H and K are two subgroups of G such that H and K are normal in G and $\mathrm{H} \cap \mathrm{K}=\{e\}$, then HK is the internal direct product of H and K .
2. Define centre of a group. Show that centre of a group is a normal subgroup of the group.
3. Define solvable group. Show that every finite abelian group is solvable.
4. Define 'Euclidean ring'. Show that 'Every field is a Euclidean ring'.
5. Define Rank and nullity of a linear transformation. Let $t: \mathrm{V}_{2}(\mathrm{R}) \rightarrow \mathrm{V}_{3}(\mathrm{R})$ defined by $t(a, b)=(a+b, a-b, b)$. Find rank and nullity of above linear transformation.
6. If K is a finite field extension of a field F and L is a finite field extension of K , then L is a finite field extension of F and $[\mathrm{L}: \mathrm{F}]=[\mathrm{L}: \mathrm{K}][\mathrm{K}: \mathrm{F}]$.
7. Let $t: \mathrm{R}^{3} \rightarrow \mathrm{R}^{3}$ be a linear transformation such that $t(a, b, c)=(3 a+c,-2 a+b,-a+2 b+4 c)$. What is the matrix of $t$ in the ordered basis $\left\{\alpha_{1}, \alpha_{2}, \alpha_{3}\right\}$, where :

$$
\begin{gathered}
\alpha_{1}=(1,0,1) \\
\alpha_{2}=(-1,2,1) \\
\alpha_{3}=(2,1,1)
\end{gathered}
$$

8. Let V be a finite dimensional vector space over a field $F$. Then the set of all eigen vectors corresponding to an eigen value $\lambda$ of a linear transformation $t: \mathrm{V} \rightarrow \mathrm{V}^{\prime}$ by adjoining zero vector to it, is a subspace of V .

## Section-C

## (Objective Type Questions)

Note : Section 'C' contains ten (10) objective type questions of one (01) mark each. All the questions of this section are compulsory.

Indicate whether the following statements are True or False :

1. Let $\mathrm{G}_{1}$ and $\mathrm{G}_{2}$ be groups, then $\mathrm{G}_{1} \times \mathrm{G}_{2} \cong \mathrm{G}_{2} \times \mathrm{G}_{1}$. (True/False)
2. $\mathrm{N}(a)$ is normal subgroup of group. (True/False)
3. An infinite abelian group does not have a composition series.
(True/False)
4. Every principal ideal domain is an 'Euclidean ring'.
(True/False)
5. Let $t: \mathrm{V} \rightarrow \mathrm{V}^{\prime}$ be a linear transformation. Then $\operatorname{ker}(t)$ is a vector subspace of V .
(True/False)
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Fill in the blanks :
6. Let V and $\mathrm{V}^{\prime}$ be vector spaces over the same field F and $t: \mathrm{V} \rightarrow \mathrm{V}^{\prime}$ be a linear transformation, then rank $(t)+\operatorname{nullity}(t)=$ $\qquad$
7. Every field of characteristic zero is $\qquad$
8. For any matrix $A$ over a field $F \operatorname{rank}\left(\mathrm{~A}^{\mathrm{T}}\right)=$ $\qquad$
9. A square matrix A of order $n$ is invertible if $\operatorname{rank}(\mathrm{A})=$
10. $A$ is a square non-singular matrix. then $\operatorname{deg}\left(\mathrm{A}^{-1}\right)=$ $\qquad$

