## MAT-501

## Advanced Algebra M.Sc. MATHEMATICS (MSCMAT-12)

First Year, Examination, 2018

## Time: $\mathbf{3}$ Hours

Max. Marks : 80
Note : This paper is of eighty (80) marks containing three (03) Sections A, B and C. Attempt the questions contained in these Sections according to the detailed instructions given therein.

## Section-A

(Long Answer Type Questions)
Note : Section 'A' contains four (04) long answer type questions of nineteen (19) marks each. Learners are required to answer two (02) questions only.

1. Every finite group $G$ has a composition series.
2. State and prove unique factorization theorem.
3. Let R be an Euclidean ring. Then any finitely generated R -Module N is the direct sum of a finite number of cyclic submodules.
4. If V and $\mathrm{v}^{\prime}$ be vector space over the same field F and $\mathrm{T}: \mathrm{V} \rightarrow \mathrm{V}^{\prime}$ be a linear transformation. If V is finite dimensional, then

$$
\operatorname{dim} \mathrm{V}=\operatorname{rank}(\mathrm{T})+\operatorname{Nullity}(\mathrm{T})
$$

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## Section-B

## (Short Answer Type Questions)

Note : Section ' $B$ ' contains eight (08) short answer type questions of eight (08) marks each. Learners are required to answer four (04) questions only.

1. Prove that the intersection of two normal subgroups of a group is a normal subgroup of group.
2. Let $G$ be finite. Show that $G$ is solvable if $G$ in $p$ solvable for all primes p .
3. Two elements $a$ and $b$ of an integral domain are associated iff one is unit times the other.
4. If an liner product space $X$ is real, show that the condition $\|x\|=\|y\|$ implies :

$$
\langle x+y, x-y\rangle=0 .
$$

5. Show that a finite field extension of prime degree is a simple extension.
6. Let H be a subgroup of all automorphisms of a field K . Then the fixed field of H is a subfield of K .
7. For any matrix $A$ over a field $F, \operatorname{rank}(A)=\operatorname{rank}\left(A^{T}\right)$.
8. For any two vectors $u$ and $v$ in an inner product space V,

$$
\|u-v\| \leq\|u\|\|v\|
$$

## Section-C

## (Objective Type Questions)

Note: Section 'C' contains ten (10) objective type questions of one (1) mark each. All the questions of this Section are compulsory.

Write True/False in the following questions :

1. External direct product and internal direct product of same factors are isomorphic.
2. The center of a group is abelian.
3. $S_{6}$ is a solvable group.
4. There are two binary operations defined in an Rmodule.
5. Each linear functional is a linear transformation.
6. The field C of complex numbers is not an algebraic extension of R .
7. Every field of characteristic zero is perfect.
8. The polynomial $x^{5}-8 x+6$ is not solvable by radical over Q.
9. Each matrix determines a linear transformation.
10. In an inner product space V , a set of orthogonal vectors is always linear dependent.
