13/1 Roll No. …………………..

MAT–501

Advanced Algebra

M. Sc. MATHEMATICS (MSCMAT–12)

First Year, Examination, 2017

Time : 3 Hours **Max. Marks : 80**

Note : This paper is of **eighty (80)** marks containing **three (03)** Sections A, B and C. Learners are required to attempt the questions contained in these Sections according to the detailed instructions given therein.

Section–A

(Long Answer Type Questions)

Note : Section 'A' contains four (04) long answer type questions of nineteen (19) marks each. Learners are required to answer *two* (02) questions only.

1. Let H and N be two subgroups of G such that N is normal in G. Then $H \cap N$ is a normal subgroup of H and $\frac{H}{H \cap N} \cong \frac{HN}{N}$.
2. Let G be a group and N < G. If N and G/N are solvable then G/N is solvable.
3. Let V be a finite dimensional vector space over the field F then there is a natural isomorphism of V onto V**.
4. State and prove Cayley-Hamilton theorem.

Section–B

(Short Answer Type Questions)

Note : Section 'B' contains eight (08) short answer type questions of eight (08) marks each. Learners are required to answer *four* (04) questions only.

1. Let G_1 and G_2 be groups, then :

$$G_1 \times G_2 = G_2 \times G_1$$

2. Let R be a Euclidean ring. A non-zero $a \in R$ is a unit iff $d(a) = d(1)$, where 1 is the unity element of R.
3. Show that a left ideal M in a ring R is an R-module.
4. If B = {(1, 0), (0, 1)} is the usual basis R^2. Determine its dual basis.
5. Let K be an extension of a field F. Then the elements of K which are algebraic over F form a subfield of K.
6. Any algebraic extension of a finite field F is a separable extension.
7. For any matrix A over a field F. Rank A = Rank A^T.
8. Any orthogonal set of non-zero vectors in an inner product space is linearly independent.

Section–C

(Objective Type Questions)

Note : Section 'C' contains ten (10) objective type questions of one (01) mark each. All the questions of this Section are compulsory.

Write True/False in the following questions.

1. External direct product and internal direct product of same factors are isomorphic.
2. The centre of a group is abelian.

3. Every ring is a Euclidean Ring.

4. There are two binary operations defined in an R module.

5. A linear transformation maps zero to zero.

Fill in the blanks in the following questions.

6. Matrix A is orthogonal then AA^T = ...

7. Let A and B be any two similar matrices over the same field F, then det A = ...

8. Being similar is an .... relation on the set of all $n \times n$ matrices having entries in the same field.

9. A field A is called .... field if all finite extensions of F are separable.

10. A linear transformation is also known as ....

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