

Roll No.

MSCMAT-12 (M.Sc. Mathematics)

First Year, Examination-2014

MAT-502

Real Analysis and Topology

Time Allowed : Three Hours

Maximum Marks : 60

Note : This paper is of sixty (60) marks divided into three (03) sections. Learners are required to attempt the questions contained in these sections according to the detailed instructions given therein.

Section - A

(Long answer type Questions)

Note : Section 'A' contains four (04) long-answer-type questions of fifteen (15) marks each. Learners are required to answer any two (02) questions only. **(2×15=30)**

1. If A and B are two sets such that $A \subset B$, then $m^*(A) \leq m^*(B)$ and for every singleton set A

$$m^*(A) = 0$$

2. Show that a continuous function defined on a measurable set is always measurable. However its converse is not always true.

3. Define subspace of a topological space Let (X, T) be a topological space and $Y \subset X, Y \neq \emptyset$
 Let $S = \{G \cap Y : G \in T\}$
 Show that S is a topology on Y
4. Define T_1 and T_2 Axioms of separation show that every T_2 -space is a T_1 -space but the converse is not true.

Section - B

(Short answer type Questions)

Note : Section 'B' contains eight (08) short-answer-type questions of five (5) marks each. Learners are required to answer any four (04) questions only. (4×5=20)

1. Show that the outer measure of an interval is its length.
2. Define 'Measurable Sets' Show that if E is a set such that $m^*(E) = 0$, then E is measurable. \Rightarrow
3. Define Measurable functions Let f and g are measurable functions defined on a measurable set E then $f + g$ is also measurable.
4. Define Lebesgue integral of a bounded function show that

$$\int_E kf = k \int_E f$$
 where $k > 0$ and $k \in \mathbb{R}$
5. Define usual topology on \mathbb{R} where \mathbb{R} is the set of real numbers.
6. Define closure of a subset of a topological space show that

$$\overline{(A \cup B)} = \overline{A} \cup \overline{B}$$
7. Show that a subset of a topological space is open iff it is nbd of each of its point.

8. Let $X = \{1, 2, 3, 4\}$

and $A = \{ \{1, 2\}, \{2, 4\}, \{3\} \}$

Determine the topology on X generated by the elements of A and hence determine the base for this topology.

Section - C

(Objective type Questions)

Note : Section 'C' contains ten (10) objective-type questions of one (01) mark each. All the questions of this section are compulsory. (10×1=10)

1. For every set A and for each $x \in \mathbb{R}$ $m^*(A + x)$ is equal to the

2. If $P = \{E_1, E_2, \dots, E_n\}$ is a measurable partition of the

\Rightarrow

measurable set E , then it is necessary that $\sum_{i=1}^n E_i = E$ and

3. Two functions f and g in $L_2[a, b]$ are said to be orthogonal on the closed interval $[a, b]$ if

4. Let X be any set and $P(X)$ be the power set of X , then $P(X)$ is a topology on X called

5. A topological space (X, T) is said to be a door space if

Choose the correct alternative :

6. A subset B of X , in (X, T) topological space is called dense subset of X if :

(a) $\overline{B} = B$

(b) $\overline{B} = \phi$

- (c) $\overline{B} = X$
 - (d) None
7. Every singleton set in a T_2 -Space is :
- (a) Open
 - (b) Closed
 - (c) Both open and closed
 - (d) None
8. If U is usual topology on \mathbb{R} then the space (\mathbb{R}, U) is :
- (a) Compact
 - (b) Not compact
 - (c) Sometimes compact Sometimes not compact
 - (d) None
9. If A and B are any two disjoint subset of \mathbb{R} $m^*(A \cup B)$ is
- (a) Less than $m^*(A) + m^*(B)$
 - (b) Greater than $m^*(A) + m^*(B)$
 - (c) Zero
 - (d) Equal to $m^*(A) + m^*(B)$
10. The closed interval $[0, 1]$ is :
- (a) Countable
 - (b) Uncountable
 - (c) A finite set
 - (d) None