

Roll No.....

BCA -10(Bachelor of Computer Application)

Second Semester, Examination-2012

BCA – 205

Mathematics II

Time: 3 Hours

Max Marks: 60

Note: The Question paper is divided into three sections A, B and C. Answer the questions as per instructions given in each section.

Section –A

(Long Answer's Question)

Note: Answer any two questions. Each question carries 15 marks.

$2 \times 15=30$

1. Prove that the greatest integer function $[x]$ is continuous at all points except at Integer points.
2. Find the area bounded by the curve $x^2=4y$ and the straight line $x=4y-2$.
3. Prove that a sequence of real numbers converges if and only if it is a Cauchy Sequence.
4. Show that the circle on the chord $x \cos \alpha + y \sin \alpha = p$ of the circle $x^2 + y^2 = a^2$ as Diameter is $x^2 + y^2 - a^2 - 2p(x \cos \alpha + y \sin \alpha - p) = 0$.

Section – B

(Short Answer's Question)

Note: Answer any four questions. Each question carries 5 marks. $4 \times 5=20$

1. Evaluate $\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{\left(\frac{\pi}{2} - x\right)^2}$

2. Discuss the continuity of the function $f(x)$ at $x = 2$

$$f(x) = \begin{cases} 2-x, & x < 2 \\ 2+x, & x \geq 2 \end{cases}$$

3. Evaluate $\int \frac{4(\sin^{-1} x)^3}{\sqrt{1-x^2}} dx$

4. Evaluate $\int \frac{\sin^{-1} x}{(1-x)^{3/2}} dx$

5. Show that the sequence $\langle S_n \rangle$, where

$$S_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} \dots \dots \dots + \frac{1}{n+n} , \text{ is convergent.}$$

6. Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

7. For what value of k will the straight line $3x+4y = k$ touch the circle?

$$x^2 + y^2 = 10x.$$

8. Prove that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$.

Section – C

Objective Question (Compulsory)

Note: Answer all questions. Each question carries 1 mark. $10 \times 1 = 10$

Note: Write True/False against the following-

1. Every constant function is continuous everywhere (True / False)
2. Every convergent sequence has a unique limit. (True/ False)
3. The equation of the straight line which cuts off intercepts a and b respectively from

the x and y - axis is $\frac{x}{a} + \frac{y}{b} = 1$. (True/ False).

4. Definite integration satisfies the property $\int_a^b f(x)dx = \int_a^b f(t)dt$.

(True/ False)

5. The value of e lie between 2 and 3.

(True/ False).

Choose the correct alternative.

6. If $n \in \mathbb{Q}$, then $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a}$ is:

- (a) $(n-1)a^n$ (b) na^n (c) na^{n-1} (d) None of these.

7. The value of $\int_0^{\pi/4} \tan^2 x \, dx$ is:

- (a). $\pi/4$ (b). $(\pi/4)^2$ (c). $(1-\pi/4)$
(d). $\pi/2$

(8) The distance between the line $12x - 5y + 9 = 0$ and the point $(2, 1)$ is:

- (a) $13/28$ (b) $28/13$ (c) $14/27$ (d) $23/14$

(9) For any sequence $\{a_n\}$ and $\{b_n\}$ of real numbers. We always have

- (a). $\lim_{x \rightarrow \infty} \text{Sup}(a_n + b_n) = \lim_{x \rightarrow \infty} \text{Sup}a_n + \lim_{x \rightarrow \infty} \text{Sup}b_n$
(b). $\lim_{x \rightarrow \infty} \text{Sup}(a_n + b_n) \leq \lim_{x \rightarrow \infty} \text{Sup}a_n + \lim_{x \rightarrow \infty} \text{Sup}b_n$
(c). $\lim_{x \rightarrow \infty} \text{Sup}(a_n + b_n) \geq \lim_{x \rightarrow \infty} \text{Sup}a_n + \lim_{x \rightarrow \infty} \text{Sup}b_n$
(d). None of the above.

10. The equation of the circle whose center is $(2, -3)$ and radius is 8 will be:

- (a). $x^2 + y^2 - 4x + 6y - 51 = 0$
(b). $x^2 - y^2 + 4x + 6y - 51 = 0$
(c). $x^2 + y^2 - 4x - 6y + 51 = 0$
(d). None of these.