Roll No. …………………..

MAT–509

Integral Transforms and Integral Equations

M. Sc. MATHEMATICS (MSCMAT–12)

Second Year, Examination, 2017

Time : 3 Hours **Max. Marks : 60**

Note : This paper is of **Sixty (60)** marks containing **three (03)** sections A, B and C. Learners are required to attempt the questions contained in these sections according to the detailed instructions given therein.

Section–A

(Long Answer Type Questions)

Note : Section 'A' contains four (04) long answer type questions of fifteen (15) marks each. Learners are required to answer *two* (02) questions only.

1. Reduce the differential equation :

$$\phi''(x) - 3\phi'(x) + 2\phi(x) = 4\sin x$$

with the conditions

$\phi(0) = 1, \quad \phi'(0) = -2,$ into a non-homogenous Volterra's integral equation of second kind.

2. Solve $(D^2 + 9)y = \cos 2t$ if $y(0) = 1,\ y\left(\frac{\pi}{2}\right) = -1$ and $D \equiv \frac{d}{dx}$.

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3. Find the Fourier sine and cosine transform of e^{-x} and using the inversion formulae, recover the original functions in both cases.

4. If $f^{(n)}(x)$ is the nth derivative of $f(x)$ with respect to x, then prove that :

$$M[f^{(n)}(x); P] = \frac{(-1)^n \Gamma P}{\Gamma P - n} M[f(P-n)]$$

Section–B

(Short Answer Type Questions)

Note : Section 'B' contains eight (08) short answer type questions of five (05) marks each. Learners are required to answer *four* (04) questions only.

1. Prove that : $\phi(x) = (1 + x^2)^{-3/2}$ is a solution of Volterra integral equation :

$$\phi(x) = \frac{1}{1+x^2} - \int_0^x \left(\frac{t}{1+t^2}\right)\phi(t)\, dt$$

2. Form an integral equation corresponding to the differential equation :

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0,$$

with the initial conditions $y(0) = 1,\ y'(0) = 0$.

3. Find the resolvent kernel of integral equation $\phi''(x) - 2\phi'(x) + \phi(x) = 0$; $\phi(t) = 0$, $\phi'(t) = 1$ if $k(x, t) = 2 - (x - t)$, $\lambda = 1$.

4. State and prove second shifting theorem for Laplace transform.

5. For Mellin transform, prove that :

$$M\left[\frac{1}{x} f\left(\frac{1}{x}\right); P\right] = M \quad f(1-P)$$

6. Find the Hankel transform of $\frac{df}{dx}$ when $f = \frac{e^{-ax}}{x}$ and $n = 1$.

7. Find :

$$L^{-1}\left\{\frac{e^{4-3P}}{(P+4)^{5/2}}\right\}$$

8. Evaluate :

$$\int_0^\infty \left(\frac{e^{-at} - e^{-bt}}{t}\right) dt$$

Section–C

(Objective Type Questions)

Note : Section 'C' contains ten (10) objective type questions of one (01) mark each. All the questions of this section are compulsory.

1. An integral equation in Fredholm type if :
   (i) Both the limits are constants
   (ii) Both the limits are variable
   (iii) Only upper limit is variable
   (iv) Only lower limit is variable

2. In integral equations, G (x, t) is called :
   (i) Fredholm's function

(ii) Green's function

(iii) Volterra's function

(iv) None of these

3. Equation, which and unknown function appears under the integral sign is called as :

(i) Linear differential equation

(ii) Partial integral equation

(iii) Linear integral equation

(iv) Gaussian integral equation

4. The solution, corresponding to eigen values of λ, can be expressed as :

(i) Sum of eigen functions

(ii) Arbitrary multiples of eigen functions

(iii) Difference of eigen functions

(iv) None of these

5. $\mathrm{L}\{\sinh at\}$ is, $\mathrm{P} > |a|$:

(i) $\dfrac{a}{\mathrm{P}^2 + a^2}$

(ii) $\dfrac{\mathrm{P}}{\mathrm{P}^2 + a^2}$

(iii) $\dfrac{\mathrm{P}}{\mathrm{P}^2 - a^2}$

(iv) None of these

6. A kernel K (x, t) of Fredholm integral equation is said to be ...... if it can be expressed as the sum of a finite number of terms, each of which is the product of a function of x alone and a function of t alone :

   (i) Non-degenerate

   (ii) Separable

   (iii) Non-separable

   (iv) Degenerate

7. Hankel transform of $\left\{\dfrac{e^{-x}}{x}\right\}$ is :

   (i) $(1 + P^2)^{-1/2}$

   (ii) $(1 + P)^{-1/2}$

   (iii) $(1 + P^2)^{-3/2}$

   (iv) None of these

8. In Mellin transform, $M[x^a f(x); P] =$:

   (i) M $f(x - a)$

   (ii) M $f(x / a)$

   (iii) M $f(x + a)$

   (iv) None of these

9. $L\{t^2 e^{2t}\}$ is :

   (i) $\dfrac{2}{(P-2)^2}$

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(ii) $\dfrac{P}{(P-2)^3}$

(iii) $\dfrac{2}{(P-2)^3}$

(iv) $\dfrac{2}{(P+2)^3}$

10. $L\left(\dfrac{e^{at}-1}{a}\right)$ is, $P > 0$:

(i) $\dfrac{1}{P\,(P+a)}$

(ii) $\dfrac{1}{P\,(P-a)}$

(iii) $\dfrac{P}{P-a}$

(iv) None of these

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