Roll No. ....

MSCMAT-12 (M.Sc. Mathematics)
First Year Examination-2015
MAT-503
Differential Equations, Calculus of Variations and Special Functions

Time : 3 Hours **Maximum Marks :** 60

Note : The Question paper is divided into three section A, B and C. Attempt Questions of each section according to given instruction.

Section - A

(Long Answer Type Questions)

Note : Answer any two questions. All questions carries equal marks. (2×15=30)

1. Solve $t + s + q = 0$

2. Prove that $J_0^2 + 2\left(J_1^2 + J_2^2 + J_3^2 + ...\right) = 1$

3. Solve $z(z - y)dx + (z + x)\, zdy + x(x + y)\, dz = 0$

4. Prove that $\int_0^\infty e^{-x} L_n(x) L_m(x) dx = \delta_{mn}$

Section - B

(Short Answer Type Questions)

Note : Answer any four (04) questions. Each question carries equal marks. (4×5=20)

1. Solve

$$(yz + xyz)dx + (zx + xyz)dy + (xy + xyz)dz = 0$$

2. Solve

$$ys + p = \cos(x + y) - y \sin(x + y)$$

3. Show that

$$\frac{d}{dx} F(\alpha; \beta; \gamma; x) = \frac{\alpha\beta}{\gamma} F(\alpha + 1, \beta + 1; \nu + 1; x)$$

4. Evaluate $\int_{-\infty}^{\infty} x e^{-x^2} H_n(x) H_m(x)\, dx$

5. Show that $\int_0^{\infty} x^{-n} J_{n+1}(x)\, dx = \frac{1}{2^n \Gamma(n+1)}, \quad n > \frac{-1}{2}$

6. Prove that

$$H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$$

7. Solve

$pt - qs = q^3$

8. Solve

$(yz + z^2)dx - xzdy + xydz = 0$

Section - C

(Objective Type Questions)

Note : Section 'C' contains ten (10) objective-type questions of ½ mark each. All the questions of this section are compulsory. (10×1=10)

1. $H_0^1(x) =$ ....

2. $|J_0(x)| \leq$ ....

3. $L_n^1(o) =$ ....

4. $F(\alpha, \beta, \gamma; 1) =$ ....

5. ${}_1F_1(\alpha; \alpha; x) =$ ....

6. $P_n(-1) =$ ....

7. $F(\alpha; \beta; x) =$ ....

8. $f(x; y; z; p; q) = 0$ is a .... equation of the .... order.

9. The diff equation

$(1 - x^2)\ y'' - 2xy^1 + n(n + 1)\ y = 0$ is called Legendre's equation, if n is ....

10. $Pdx + Qdy + Rdz = 0$, where P,Q,R are equations of x, y and z is called a ....