Roll NO -----------------------

MSCMAT-12 (**M.Sc. MATHEMATICS**)
First Year, Examination-2013

MAT-501
Advanced Algebra

Time: 3 Hours Max. Marks: 60

Note: The question paper is divided in to three sections 'A' 'B' and 'C'. Attempt question of each section according to given instructions.

Section – A
Long Answer Questions

Note: Answer any two questions. Each question carries 15 marks. 2X15=30

1. Prove that a purely transcendental extension is totally transcendental.
2. Let F $\underline{\subset}$ E with $|E : F| < \infty$. Then prove that following are equivalent.

   (i) E is a Galois extension of F

   (ii) E is both reparable and normal over F.

   (iii) E is splitting with field over F for some reparable polynomial over F.
3. Let G be a finite abelian p-group and $C \underline{\subset} G$ be cyclic subgroup with maximum possible order. Then prove that $C = C \dot{X} B$ for some subgroup B $\underline{\subset}$ G.
4. Let U be finitely generated right R-module.

   If UJ(R) = U, then Prove that U=0.

Section – B
Short Answer Questions

Note: Answer any Four Questions. Each question carries 05 marks. 5X4=20

1. Prove that a group G is solvable if and only if $G^{(n)} = 1$ for some n.
2. Let $0 \neq f \in F[X]$. Then prove that there exists $E \supseteq F$ such that f splits over E.
3. Let G be finite. Show that G is solvable if G in p- solvable for all primes p.
4. Le $p_1, p_2, p_3, \ldots \ldots \ldots .. p_n$ be different prime numbers and let

   $E = Q\left[\sqrt{p_1}, \sqrt{p_2}, \ldots \ldots \ldots .. \sqrt{p_n}\right]$ in R.

   Then show that E is Galois over Q.
5. Prove that the sub modules of the R-module R^1 are the ideals of R.
6. If an liner product space X is real, show that the condition $\|x\| = \|y\|$ implies

   $< x + y , x - y = 0$

Explain it geometrically.

7. Let X, Y be vector spaces, both real or both complex

   Let $T: D(C) \rightarrow X$ be a linear operator with domain $D(T) \subset X$ and rang $R(T) \subset Y$. Then prove that if T^{-1} exists, it is linear operator.

8. If in an inner product space, $x_n \rightarrow x$ and $y_n \rightarrow y$, then prove that

$$< x_n, y_n \rightarrow < x, y >$$

Section – C

Objective Questions

Note: Answer all questions. Each question carries 01 mark **10X1=10**

Define the following terms:

1. Euclidean Ring.
2. Commutators
3. Rank of a matrix
4. Splitting fields
5. Solvable group
6. Right R- module
7. Inner product space
8. Derived subgroups
9. Homomorphism of fields
10. Euclidean Domain.